Worksheet 1.

In this worksheet, we will explore functions, function notation, and a specific type of function- the line!

Examples: $f(x) = 2x + 3, \quad g(x) = \sin x, \quad s(t) = e^t$

One of the simplest functions we can work with is a linear function (i.e. a line).

Lines To determine a line, we only need a point and a slope. You can think of the point as a starting place for your line, and the slope as determining where all the other points on your line go.

- Slope $= m = \dfrac{\text{rise}}{\text{run}}$
- Given any two points on your line $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

The same line can be expressed with different equations, but in the end they are all equivalent.

- Slope-intercept form: $y = mx + b$ where m is the slope of the line and $(0, b)$ is the y-intercept.
- Point-slope form: Given a point (x_1, y_1) on the line with slope m, $y - y_1 = m(x - x_1)$
- Standard form: $Ax + By = C$ where slope $m = -A/B$ and $(0, C/B)$ is the y-intercept

Try these:

1. Find an equation of a line with slope $m = 2$ passing through the point $(-3, 5)$.
2. Find an equation of a line passing through the points $(-4, 7)$ and $(6, -9)$.
3. Does the point $(-2, 1)$ lie on the line $y = 3x + 7$? Why or why not?
4. Find an equation of a line passing through the point $(11, 2)$ that is parallel to the line $3x - 4y = 5$.
5. Are the lines $y = \dfrac{2}{3}x + 6$ and $y - 3 = \dfrac{3}{2}(x - 1)$ parallel, perpendicular, or neither?

Extra Practice:

1. Find an equation of a line with slope $m = -3/4$ passing through the point $(6, 2)$.
2. Find an equation of the line passing through the points $(2, \frac{1}{2})$ and $(\frac{1}{2}, 2)$
3. Find an equation of a line passing through the point $(-3, 2)$ that is perpendicular to the line $x + y = 9$

Functions When you have an equation of a line (in any form), you should be able to solve for y in terms of x. When we can do this, we can think of x as the independent variable (the input of the function that we get to choose), y as the dependent variable (the output of the function that depends on x), and the resulting equation as expressing y as a function of x.

In general, we can think of a **function** as a relation between two sets, the domain (input) and range (output). Often, this relation is given to us as an expression or formula. This formula tells us what to do to the input to get an output.

Examples:

1. If $y = 2x + 3$, we want to multiply every input, x, by 2, then add 3 to get our output. If our input is $x = 5$ then we see that $y = 2(5) + 3 = 10 + 3 = 13$.

2. If $f(x) = x^2 - 7$, we want to square every input, x, then subtract 7 to get our output. If our input is $x = 6$ then we get $g(6) = 6^2 - 7 = 36 - 7 = 29$.

3. If $s(t) = t - 3t^2$, we want to take the input t and subtract from it 3 times t squared. If our input is $t = -2$, then we get $h(-2) = (-2) - 3(-2)^2 = 2 - 3(4) = (-2) - 12 = -14$

Note

- We often use x for our input (independent) variable and y for our output (dependent) variable, but it does not always have to be so!

- Above, we were using *function notation*, which emphasizes the independent variable as well as gives a name to the dependent variable.

More on function notation:

1. $f(x) = x - 3x^2$: The output is denoted $g(x)$, read as "g of x, where x is the input.

2. $s(t) = t - 3t^2$: The output is denoted $s(t)$, read s "s of t", where t is the input.

Try these:

1. If $g(x) = x^2 - 3x + 2$, evaluate and simplify $g(0)$, $g(3)$, and $g(x + h)$.

2. If $s(t) = \frac{t+2}{t-1}$, evaluate and simplify $s(-2)$, $s(1)$, and $s(t + h)$.

3. If $f(x) = 3x^2 - x$, evaluate and simplify $f(a + h) - f(a)$?.

Extra Practice:

1. If $V(r) = \frac{4}{3}\pi r^3$, evaluate $V(2)$, $V(1/2)$, and $V(2r)$. Note that the function $V(r) = \frac{4}{3}\pi r^3$ gives the volume of a sphere of radius r. Given this information, what does $V(2)$ represent? $V(1/2)$?

2. If $f(x) = \sqrt{x + 8} + 2$, evaluate and simplify $f(-8)$, $f(1)$, $f(x - 8)$.

3. If $g(x) = \frac{1}{x^2}$, evaluate and simplify $\dfrac{g(x) - g(5)}{x - 5}$, $x \neq 5$

Where will we see functions in calculus? EVERYWHERE! It will be important to understand functions and function notation throughout the course.

We will be talking about tangent lines when we talk about derivatives, so knowing about slope and equations of lines will be important when discussing these tangent lines.