Worksheet 2.

In this worksheet, we will explore simplifying functions and algebraic expressions. We mainly focus on factoring and expanding algebraic expressions.

<u>*Examples*</u>: $f(x) = x^2 - 4, \quad f(t) = t^3 - 3t^2 - 4t, \quad f(x) = x^3 - 2x^2 - x + 2$

<u>***Factoring Algebraic Expressions***</u>

Factoring allows us to rewrite a given complex expression as a product of its factors. When factoring polynomials, the following identities are frequently used:

- Factoring a quadratic function.
- $a^2 - b^2 = (a-b)(a+b)$ *difference between squares.*
- $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ *difference between cubes.*
- $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ *sum between cubes.*
- Factor by grouping terms.

Follow these steps when factoring:

- First, factor out any common factors. That is, if there is a common term between all the terms, factor it out, even if it's a number.
- Use any of the identities you know to factor the remaining expression.
- Continue till you can't factor any further.

<u>*Examples*</u>:

1. $f(x) = 3x^3 - 9x^2 - 30x$ First, factor out the common factor, $3x$
   $= 3x(x^2 - 3x - 10)$
   Next, factor the quadratic expression into a product of two linear factors. Here we are looking for two numbers that multiply to -10 and add up to -3.
   $f(x) = 3x(x-5)(x+2)$

2. $f(x) = x^3 - 2x^2 - x + 2$ Group the terms and takeout common factors.
   $= x^2(x-2) - (x-2)$ Factor out the common factor, $(x-2)$.
   $= (x-2)(x^2-1)$ Factor out $x^2 - 1$ using the formula $a^2 - b^2 = (a-b)(a+b)$.
   $= (x-2)(x-1)(x+1)$

3. $f(t) = t^4 - 4$ Rewrite 4 as 2^2 and then rearrange the terms in the form of $a^2 - b^2$
   $= (t^2)^2 - 2^2$ Use the formula $a^2 - b^2 = (a-b)(a+b)$
   $= (t^2-2)(t^2+2)$
   Note that $t^2 + 2$ is an irreducible quadratic factor, but we can factor $t^2 - 2$ into a product of two linear factors using the formula $a^2 - b^2 = (a-b)(a+b)$ again.
   $f(t) = (t^2 - (\sqrt{2})^2)(t^2+2)$
   $= (t - \sqrt{2})(t + \sqrt{2})(t^2 + 2)$

Try these:
Factor as much as possible.

1. $f(x) = 8x^5 + 36x^4 - 20x^3$
2. $f(t) = 2t^4 - 5t^2 - 3$
3. $g(x) = 9x^3 - 25x$
4. $h(x) = 2x^3 - 3x^2 - 4x + 6$

Extra Practice:

1. $f(x) = 15x^2 + 25x + 10$
2. $f(t) = t^6 - 27t^2$
3. $g(x) = x^5 + 3x^3 - 8x - 24$

Expanding Algebraic Expressions

When expanding polynomials, the following identities are frequently used:

- $(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$ *perfect square binomial.*
- $(a-b)^2 = (a-b)(a-b) = a^2 - 2ab + b^2$ *perfect square binomial.*
- $(a+b)^3 = (a+b)(a+b)^2 = (a+b)(a^2 + 2ab + b^2) = a^3 + 3a^2b + 3ab^2 + b^3$
- $(a-b)^3 = (a-b)(a-b)^2 = (a-b)(a^2 - 2ab + b^2) = a^3 - 3a^2b + 3ab^2 - b^3$

Examples:

1. Simplify the expression $(x-3)^2 - 5(x-3) - (x^2 - 5x)$.
   We can use the *perfect square binomial expansion*, $(a-b)^2 = a^2 - 2ab + b^2$
   $$\begin{aligned}(x-3)^2 - 5(x-3) - (x^2-5x) &= (x^2 - 6x + 9) - 5(x-3) - (x^2 - 5x)\\ &= x^2 - 6x + 9 - 5x + 15 - x^2 + 5x\\ &= -6x + 24\end{aligned}$$
2. If $g(x) = x^3 - 4x + 3$, then simplify $g(x+2) - g(x)$.
   $g(x+2) - g(x) = [(x+2)^3 - 4(x+2) + 3] - [x^3 - 4x + 3]$
   We can use $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ with $a = x$ and $b = 2$.
   $$\begin{aligned}g(x+2) - g(x) &= [(x+2)^3 - 4(x+2) + 3] - [x^3 - 4x + 3]\\ &= [(x^3 + 6x^2 + 12x + 8) - 4(x+2) + 3] - [x^3 - 4x + 3]\\ &= x^3 + 6x^2 + 12x + 8 - 4x - 8 + 3 - x^3 + 4x - 3\\ &= 6x^2 + 12x \qquad \text{We can factor out the common factor } 6x.\\ &= 6x(x+2)\end{aligned}$$

Try these:

1. If $f(x) = 2 + 5x - 3x^2$, then simplify $f(x+3) - f(x)$.
2. Simplify the expression $(x-1)^3 + 4(x-1) - (x^3 + 4x)$.
3. If $f(t) = 7 - 2t^3$, then simplify $f(a+h) - f(a)$.

Extra Practice:

1. If $f(t) = 8t - 4t^3$, then simplify $f(-2+h) - f(-2)$.
2. If $f(x) = x^2 - 2x + 1$, then simplify $f(x+5) - f(x)$.