Worksheet 2.

In this worksheet, we will explore simplifying functions and algebraic expressions. We mainly focus on factoring and expanding algebraic expressions.

**Examples:**  
\[ f(x) = x^2 - 4, \quad f(t) = t^3 - 3t^2 - 4t, \quad f(x) = x^3 - 2x^2 - x + 2 \]

**Factoring Algebraic Expressions**

Factoring allows us to rewrite a given complex expression as a product of its factors. When factoring polynomials, the following identities are frequently used:

- Factoring a quadratic function.
  
  \[ a^2 - b^2 = (a - b)(a + b) \]  
  difference between squares.

- \[ a^3 - b^3 = (a - b)(a^2 + ab + b^2) \]  
  difference between cubes.

- \[ a^3 + b^3 = (a + b)(a^2 - ab + b^2) \]  
  sum between cubes.

- Factor by grouping terms.

Follow these steps when factoring:

- First, factor out any common factors. That is, if there is a common term between all the terms, factor it out, even if it’s a number.

- Use any of the identities you know to factor the remaining expression.

- Continue till you can’t factor any further.

**Examples:**

1. \[ f(x) = 3x^3 - 9x^2 - 30x \]
   
   First, factor out the common factor, \( 3x \)
   
   \[ = 3x(x^2 - 3x - 10) \]
   
   Next, factor the quadratic expression into a product of two linear factors. Here we are looking for two numbers that multiply to \(-10\) and add up to \(-3\).
   
   \[ f(x) = 3x(x - 5)(x + 2) \]

2. \[ f(x) = x^3 - 2x^2 - x + 2 \]

   Group the terms and takeout common factors.
   
   \[ = x^2(x - 2) - (x - 2) \]
   
   Factor out the common factor, \( x - 2 \).
   
   \[ = (x - 2)(x^2 - 1) \]
   
   Factor out \( x^2 - 1 \) using the formula \( a^2 - b^2 = (a - b)(a + b) \).
   
   \[ = (x - 2)(x - 1)(x + 1) \]

3. \[ f(t) = t^4 - 4 \]

   Rewrite 4 as \( 2^2 \) and then rearrange the terms in the form of \( a^2 - b^2 \)
   
   \[ = (t^2)^2 - 2^2 \]
   
   Use the formula \( a^2 - b^2 = (a - b)(a + b) \)
   
   \[ = (t^2 - 2)(t^2 + 2) \]

   Note that \( t^2 + 2 \) is an irreducible quadratic factor, but we can factor \( t^2 - 2 \) into a product of two linear factors using the formula \( a^2 - b^2 = (a - b)(a + b) \) again.
   
   \[ f(t) = (t^2 - (\sqrt{2})^2)(t^2 + 2) \]
   
   \[ = (t - \sqrt{2})(t + \sqrt{2})(t^2 + 2) \]
Try these:
Factor as much as possible.
1. \( f(x) = 8x^5 + 36x^4 - 20x^3 \)
2. \( f(t) = 2t^4 - 5t^2 - 3 \)
3. \( g(x) = 9x^3 - 25x \)
4. \( h(x) = 2x^3 - 3x^2 - 4x + 6 \)

Extra Practice:
1. \( f(x) = 15x^2 + 25x + 10 \)
2. \( f(t) = t^6 - 27t^2 \)
3. \( g(x) = x^5 + 3x^3 - 8x - 24 \)

Expanding Algebraic Expressions
When expanding polynomials, the following identities are frequently used:

- \( (a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2 \) perfect square binomial.
- \( (a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2 \) perfect square binomial.
- \( (a + b)^3 = (a + b)(a + b)^2 = (a + b)(a^2 + 2ab + b^2) = a^3 + 3a^2b + 3ab^2 + b^3 \)
- \( (a - b)^3 = (a - b)(a - b)^2 = (a - b)(a^2 - 2ab + b^2) = a^3 - 3a^2b + 3ab^2 - b^3 \)

Examples:
1. Simplify the expression \( (x - 3)^2 - 5(x - 3) - (x^2 - 5x) \).
   We can use the perfect square binomial expansion, \( (a - b)^2 = a^2 - 2ab + b^2 \)
   \( (x - 3)^2 - 5(x - 3) - (x^2 - 5x) = (x^2 - 6x + 9) - 5(x - 3) - (x^2 - 5x) \)
   \[ = x^2 - 6x + 9 - 5x + 15 - x^2 + 5x \]
   \[ = -6x + 24 \]

2. If \( g(x) = x^3 - 4x + 3 \), then simplify \( g(x + 2) - g(x) \).
   \( g(x + 2) - g(x) = [(x + 2)^3 - 4(x + 2) + 3] - [x^3 - 4x + 3] \)
   We can use \( (a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 \) with \( a = x \) and \( b = 2 \).
   \( g(x + 2) - g(x) = [(x + 2)^3 - 4(x + 2) + 3] - [x^3 - 4x + 3] \)
   \[ = [(x^3 + 6x^2 + 12x + 8) - 4(x + 2) + 3] - [x^3 - 4x + 3] \]
   \[ = x^3 + 6x^2 + 12x + 8 - 4x - 8 + 3 - x^3 + 4x - 3 \]
   \[ = 6x^2 + 12x \] We can factor out the common factor \( 6x \).
   \[ = 6x(x + 2) \]

Try these:
1. If \( f(x) = 2 + 5x - 3x^2 \), then simplify \( f(x + 3) - f(x) \).
2. Simplify the expression \( (x - 1)^3 + 4(x - 1) - (x^3 + 4x) \).
3. If \( f(t) = 7 - 2t^3 \), then simplify \( f(a + h) - f(a) \).

Extra Practice:
1. If \( f(t) = 8t - 4t^3 \), then simplify \( f(-2 + h) - f(-2) \).
2. If \( f(x) = x^2 - 2x + 1 \), then simplify \( f(x + 5) - f(x) \).